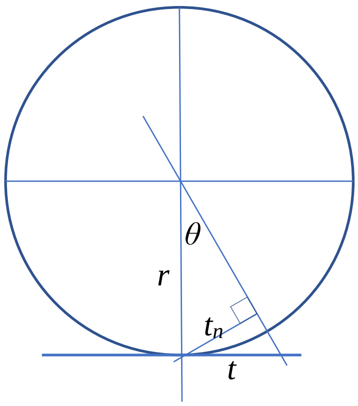
Bicycle and Motorcycle Trail

Start with the convention used in motorcycles, where the angle is measured between the steering axis and the vertical, so that the angle is interior to the triangles with sides of relevant dimensions and the trigonometry is a little easier to see.

First, without offset, where r = radius, t = trail, t_n = "normal" trail, perpendicular to the steering axis, and θ = steering axis angle measured from the vertical, as on motorcycles:



$t_n = r\sin\theta$ and

$t_n = t\cos\theta$ so combine

$r\sin\theta = t\cos\theta$ and solve for t:

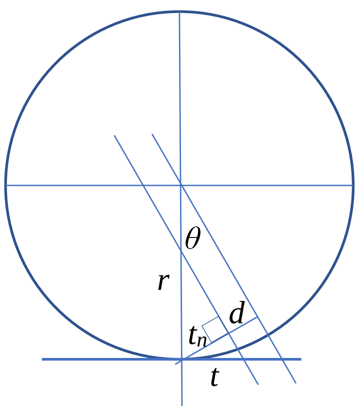
$$t = r\frac{\sin\theta}{\cos\theta} = r\tan\theta$$

It can also be seen directly that

$t = r\tan\theta$,

so all is internally consistent.

Now with perpendicular offset d from the steering axis and the wheel axle:



$t_n + d = r\sin\theta$ so

$t_n = r\sin\theta - d$ and

$t_n = t\cos\theta$ so combine

$r\sin\theta - d = t\cos\theta$ and solve for t:

$$t = \frac{r\sin\theta - d}{\cos\theta}$$

Same result as in the Bicycle and Motorcycle Geometry article.

For bicycles, where the angle is measured between the steering axis and the horizontal, simply replace θ with $90° - \theta$, and so $\sin(90° - \theta) = \cos\theta$ and $\cos(90° - \theta) = \sin\theta$.

Thus, for bicycles, $t = \dfrac{r\cos\theta - d}{\sin\theta}$. Same result as in the Bicycle and Motorcycle Geometry article.

Notice that the projection of d onto the ground, call it d_{proj}, that must be subtracted from trail without offset is longer than d, just as trail is longer than normal trail, and can be calculated as $d_{proj} = d/\sin\theta$.